MP 732, Homework 2

Due Monday Feb. 11, 2019 at 11:59 P.M. on Sakai

February 4, 2019

Problem 1, 25 pts total. Consider an x-ray tube of length l where the anode is held at potential $V = V_0$ and the cathode is held at ground, $V = 0$.

Helpful equations

$$\nabla^2 V = -\boldsymbol{\nabla} \cdot \boldsymbol{E} = -\frac{\rho}{\epsilon_0}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{I} = -\frac{\partial}{\partial x}\frac{\partial q}{\partial t} = -\frac{\partial \rho}{\partial t}$$

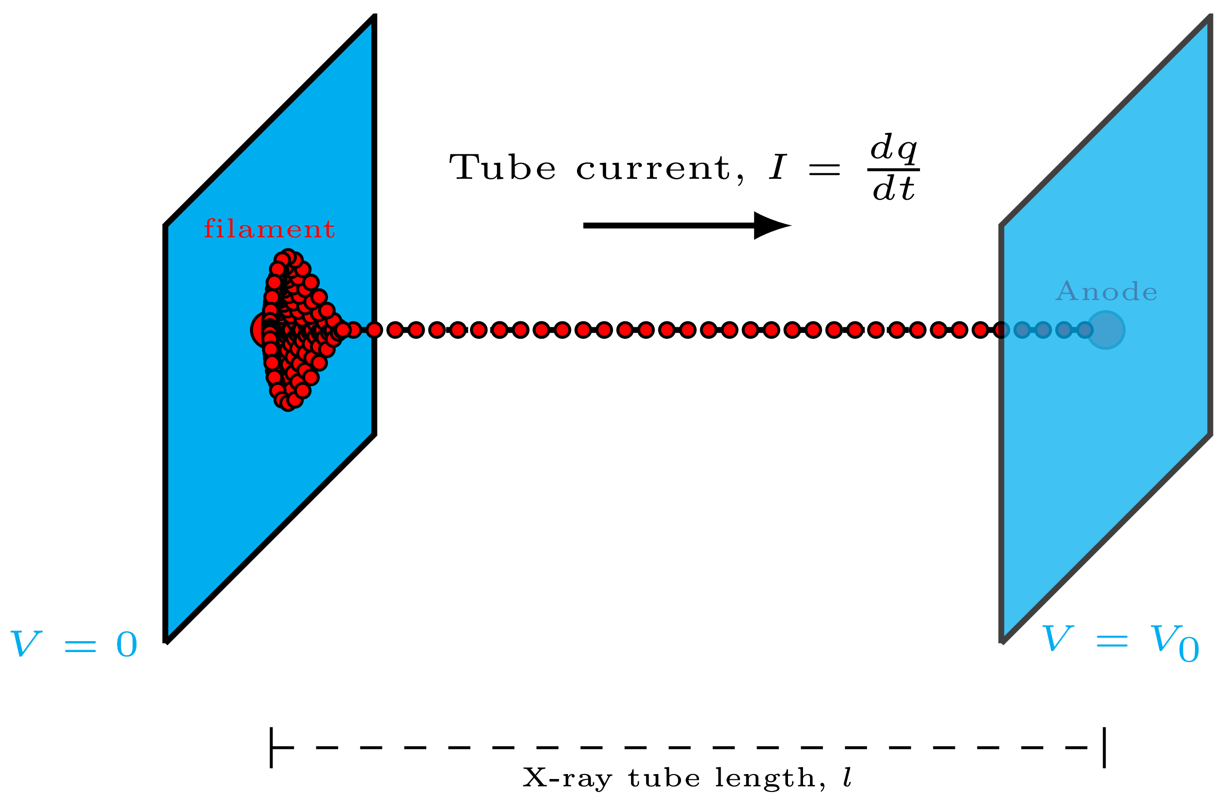

Figure 1: Space-charge–limited x-ray tube diagram.

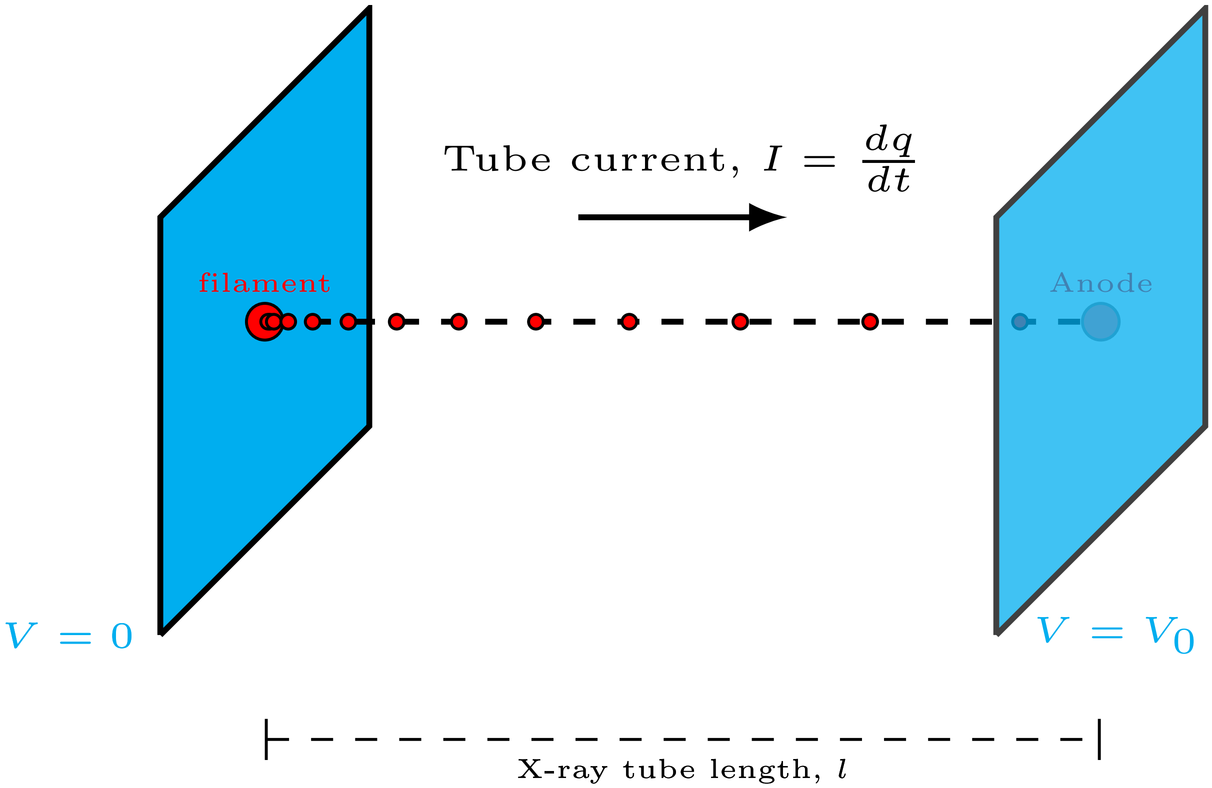

Figure 2: Emission-limited x-ray tube diagram.

Part A, 6 pts. Define and give a reasonable range of values for the x-ray tube's:
(i) filament current,
(ii) tube current,
(iii) kVp.

Part B, 8 pts. What does it mean for a tube current to be "space-charge–limited"? Give your answer in terms of your answers from Part A(i) and A(ii) and derive the relationship between the tube current, I and the kVp, V_0.

$$I \sim \frac{V_0^{3/2}}{l^2}$$

Hint:
Why is the magnitude of the electric field at the filament 0 here?
Why is $\boldsymbol{\nabla} \cdot \boldsymbol{I} = 0$ in a space-charge–limited setup, and what does this mean for the velocity of an electron passing through the tube?

Part C, 6 pts. Find another scaling relation of the form

$$I \sim {V_0}^n, n \in \mathbb{R}$$

for the emission-limited x-ray tube.
Hint:
Why is the magnitude of the electric field at the filament *not* 0 here?
Why is $\boldsymbol{\nabla} \cdot \boldsymbol{I} \neq 0$ in a space-charge–limited setup, and what does this mean for the velocity of an electron passing through the tube?

Part D, 5 pts.
(i) Sketch the solutions from B and C above on the same plot.
(ii) Approximately what kVp is necessary to go from SCL to EL tube operation (feel free to look this up, but do cite a source).
(iii) Give one example of a diagnostic imaging procedure that is typically above this threshold kVp and one typically below it.
(iv) How might a modality that operates in the kVp range associated with SCL overcome the limitation?

Problem 2, 15 pts total.

Part A, 5 pts.
Roughly sketch the bremsstrahlung spectrum intensity, I, produced by electrons with kinetic energy 100 keV striking a thick Tungsten target:
(i) Without filtration
(ii) With filtration (any reasonable filter is fine)

Part B, 5 pts.
Why is the answer to part A(i) approximately linear? What is the slope of the line?

Part C, 5 pts.
Explain briefly the steps below that allow us to arrive at a relationship between the bremsstrahlung spectrum intensity and the kVp & Z.

$$
\begin{aligned}
I_\nu &\propto Z\left(h\nu_{\text{max}} - h\nu\right) \\
I &\propto Z\int_0^{h\nu_{\text{max}}} \left(h\nu_{\text{max}} - h\nu\right) dh\nu \\
&\propto \frac{1}{2} Z\left(h\nu_{\text{max}}\right)^2 \\
&\propto \frac{1}{2} Z\left(\text{kVp}\right)^2
\end{aligned}
$$