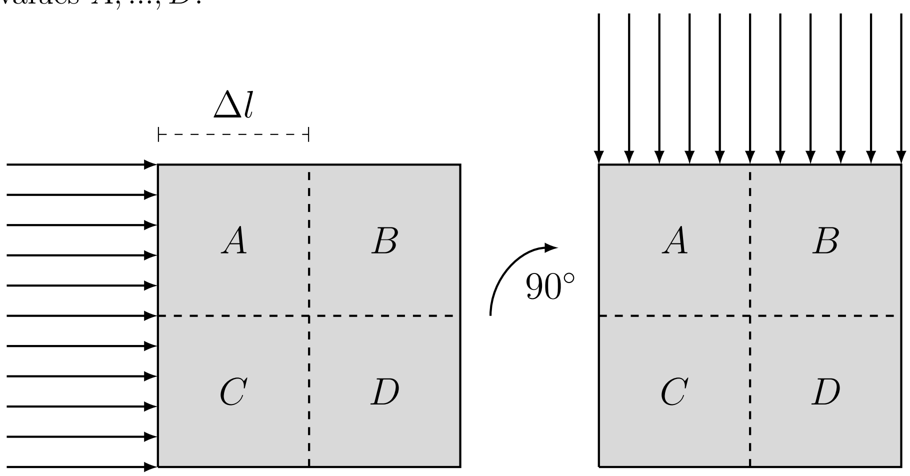
MP 732, CT Lecture 1 Quiz

Due Wed March 13th, 2019 at 11:59 PM on Sakai

May 11, 2020

Problem 1. Consider the 2D, four-material object shown below with ground-truth linear attenuation values $A, ..., D$.



Part A. Using the two projection angles illustrated, write the line integrals

$$\int \mu \cdot d\vec{l} \approx \Delta l \sum \mu_{i,j}$$

that would be the observed measurement in each case.

Part B. Use one iteration of backprojection (i.e., 'smear' the measurements back across both projection directions) starting with an initial estimate of

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

to reconstruct the original object given the result from **A**. How well does backprojection compare to the ground-truth? Under what conditions would backprojection as a reconstruction method give potentially misleading results?

Part C. Using the unknowns $A, ..., D$ as the vector V, and the results from **A** as the vector W, write the corresponding system of equations in the form:

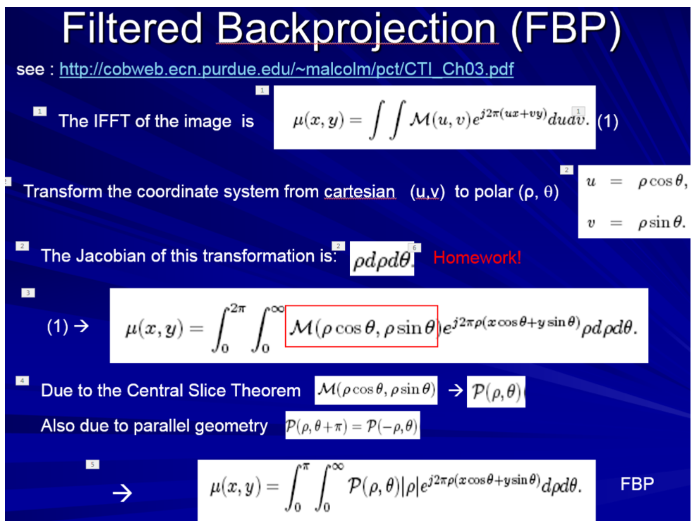
$$U_{n \times n} V_{n \times 1} = W_{n \times 1}.$$

Part D.[1] Determine the left inverse of U. Is it a true inverse? If so, what is the consequence of U being invertible? If not, what is the consequence of U being only approximately invertible? What then is the value of

$$V_{\text{matrix inversion}} = U^{-1}W?$$

How does it compare to the ground-truth? How does it compare to the result from simple backprojection? Under what conditions would matrix inversion as a reconstruction method give potentially misleading results?

Problem 2. As indicated in slide 29 from the March 5^{th} lecture, you will show that the Jacobian of the transformation from cartesian coordinates (u, v) to polar coordinates (ρ, θ) is $\rho d\rho d\theta$.



Lecture 1, slide 29 (courtesy of Dr. Badea)

Given that

$$u = \rho\cos\theta$$
$$v = \rho\sin\theta$$

prove that the Jacobian

$$J = \begin{vmatrix} \dfrac{\partial u}{\partial \rho} & \dfrac{\partial u}{\partial \theta} \\ \dfrac{\partial v}{\partial \rho} & \dfrac{\partial v}{\partial \theta} \end{vmatrix} = \rho d\rho d\theta.$$

Additionally, comment on the significance of ρ. Explain what you would expect its effect on the resulting reconstruction to be.

[1] The MATLAB symbolic toolbox (declared in code as `syms var1 var2 var3 var4`) as well as the function `pinv` (pseudo-inverse) may be useful here.